Rivelatori X e Gamma per spettroscopia e imaging

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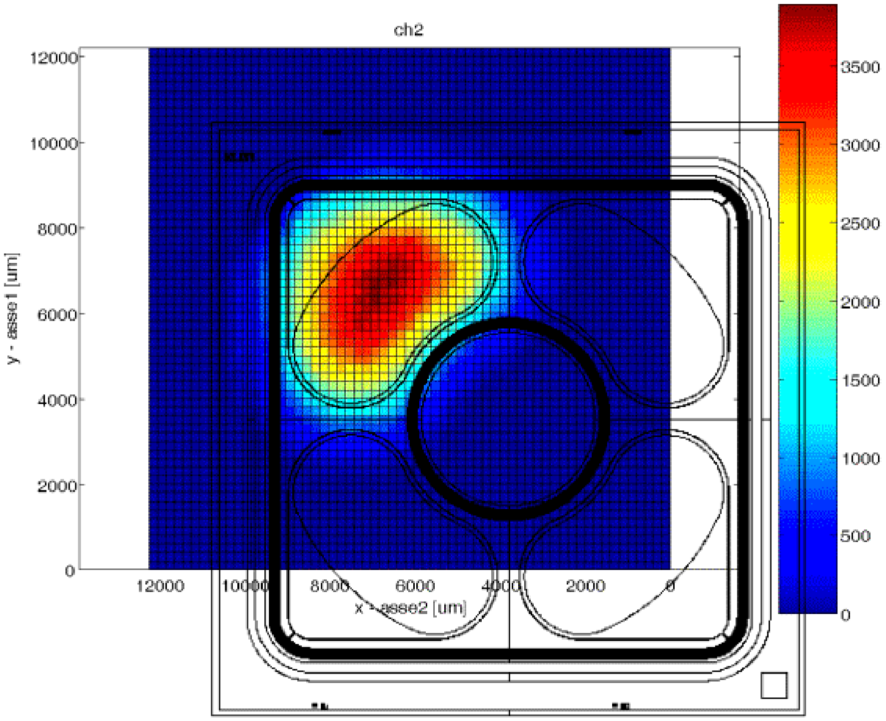


Scuola Nazionale INFN
“Rivelatori ed Elettronica per Fisica
delle Alte Energie, Astrofisica
ed Applicazioni Spaziali“,
Laboratori Nazionali di Legnaro
4-8 Aprile 2005

Summary

First part

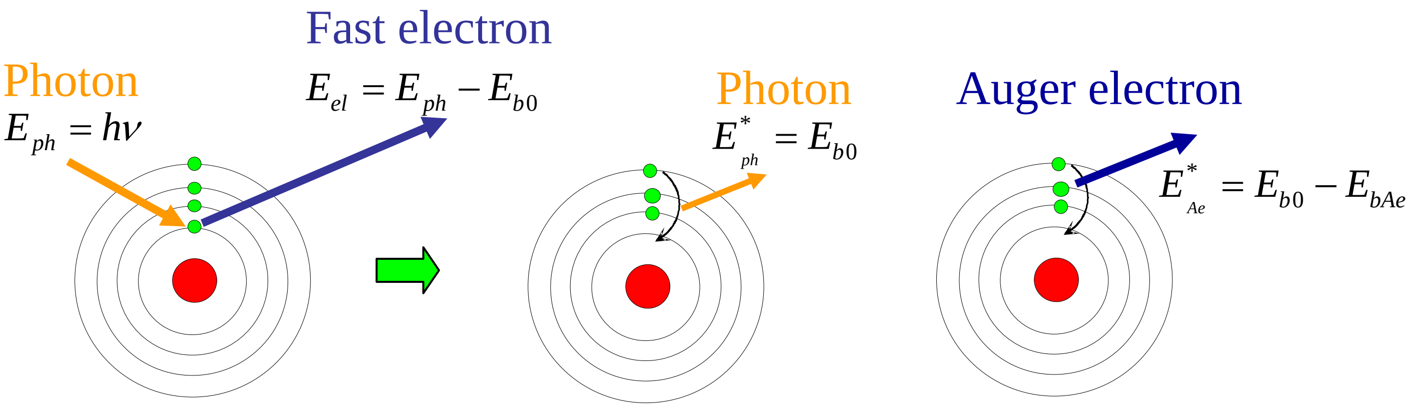
- why “low capacitance” for spectroscopic detectors (in other words, why SDDs?): some basic concepts on signal processing

Second part

- Introduction to some new SDDs and SDD monolithic arrays
- Some applications in material analysis, in biomedicine and in atomic physics

The interaction of the radiation with the detector

The main interactions of X rays in semiconductor detectors



Photon

$E_{ph} = h\nu$

Fast electron

$E_{el} = E_{ph} - E_{b0}$

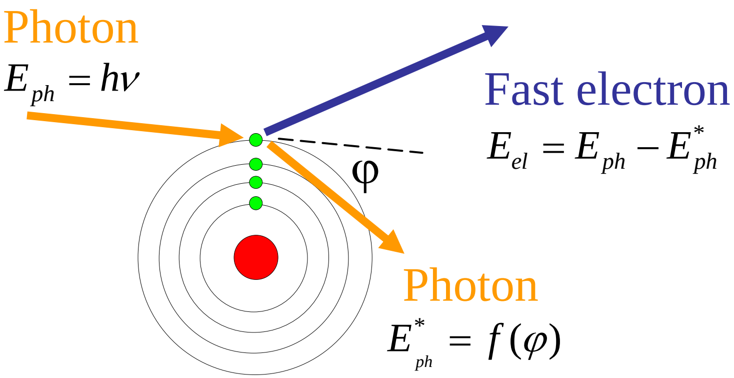
Photon

$E^{*}_{ph} = E_{b0}$

Auger electron

$E^{*}_{Ae} = E_{b0} - E_{bAe}$

Photoelectric absorption



Photon

$E_{ph} = h\nu$

Fast electron

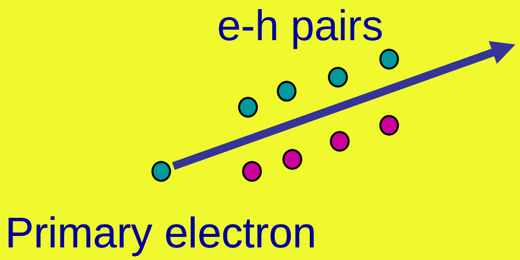
$E_{el} = E_{ph} - E^{*}_{ph}$

φ

Photon

$E^{*}_{ph} = f(\varphi)$

Compton scattering

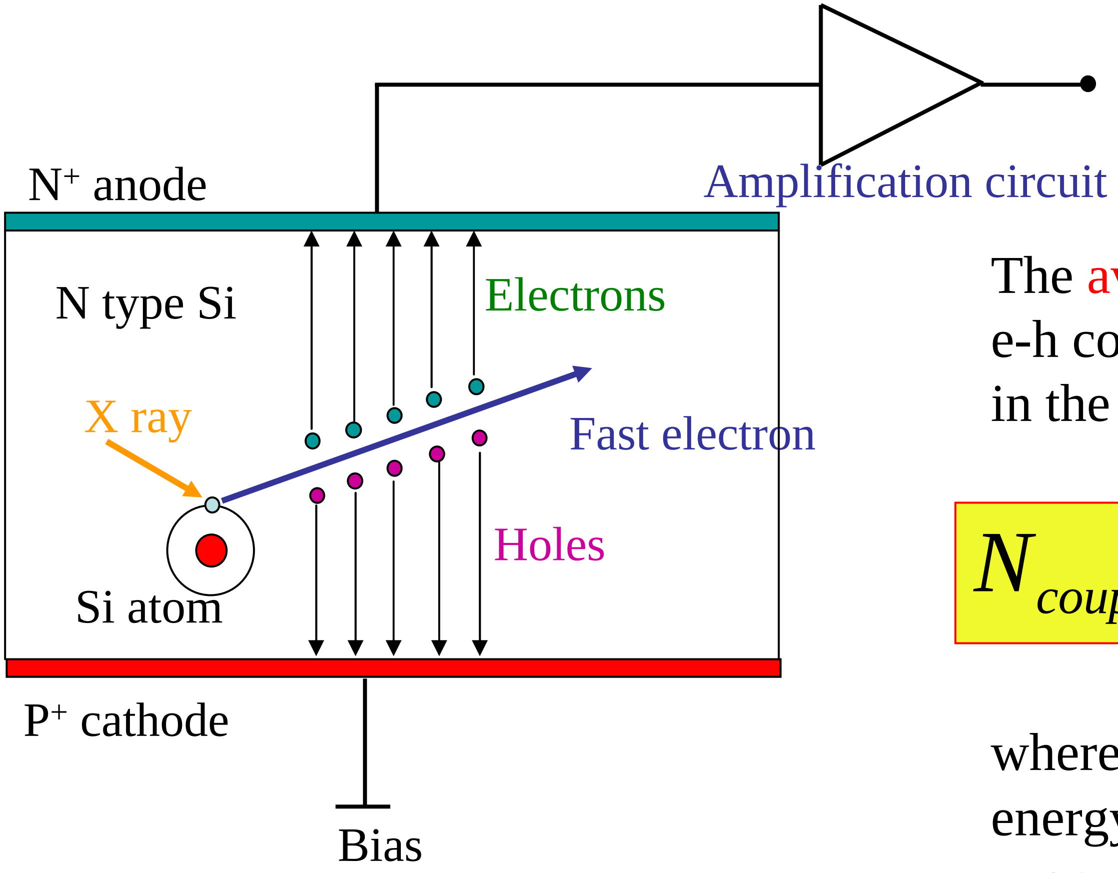


e-h pairs

Primary electron

Electron-hole pairs generation

Signal generation in a detector

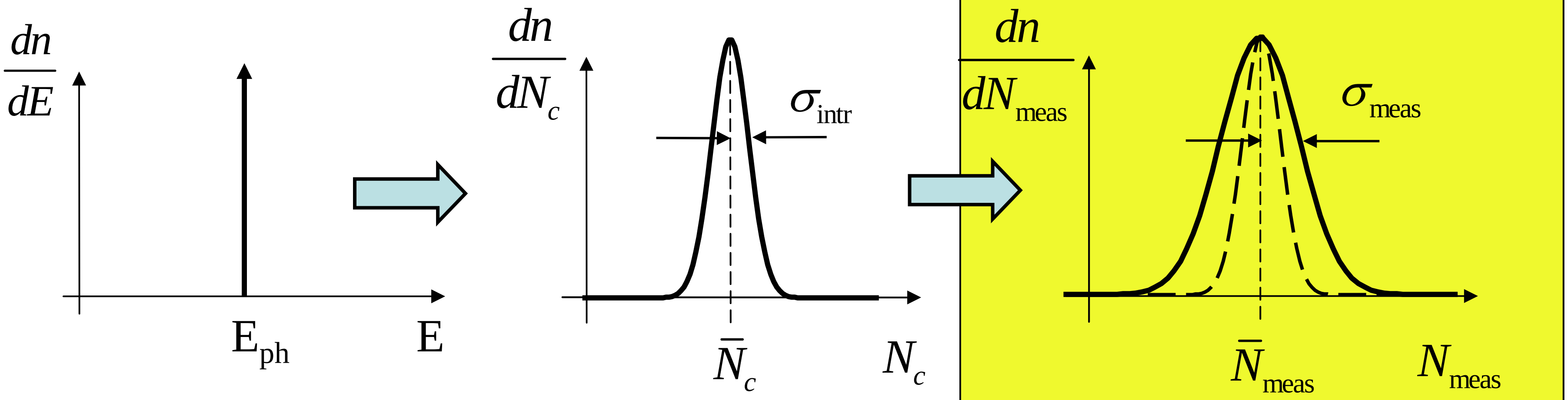


The average number of e-h couples generated in the detector is:

$$N_{coup} = E_{ph} / \varepsilon_{coup}$$

where E_{ph} is the photon energy and ε_{coup} is the average energy required to generate an e-h couple (ε_{coup}= 3.6 eV in Si).

The generation of electron-hole couples is a statistical process, the electronic noise introduces further statistical fluctuations



Incident photon.
monoenergetic

Statistical spreading of the number of couples e-h

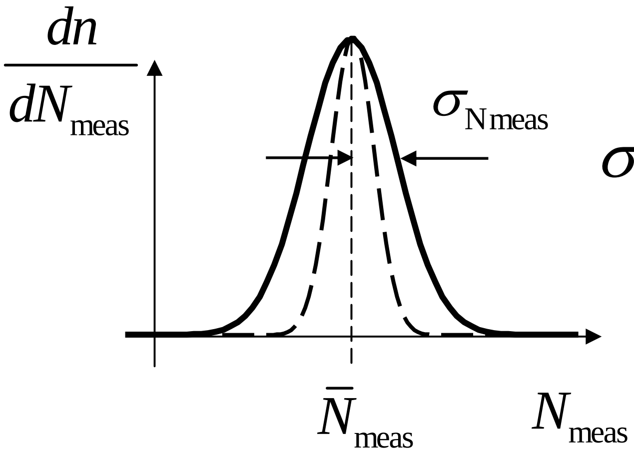
Electronic noise.
contribution

$$\sigma^2_{N_c} = F \cdot E_{ph} / \varepsilon_c$$

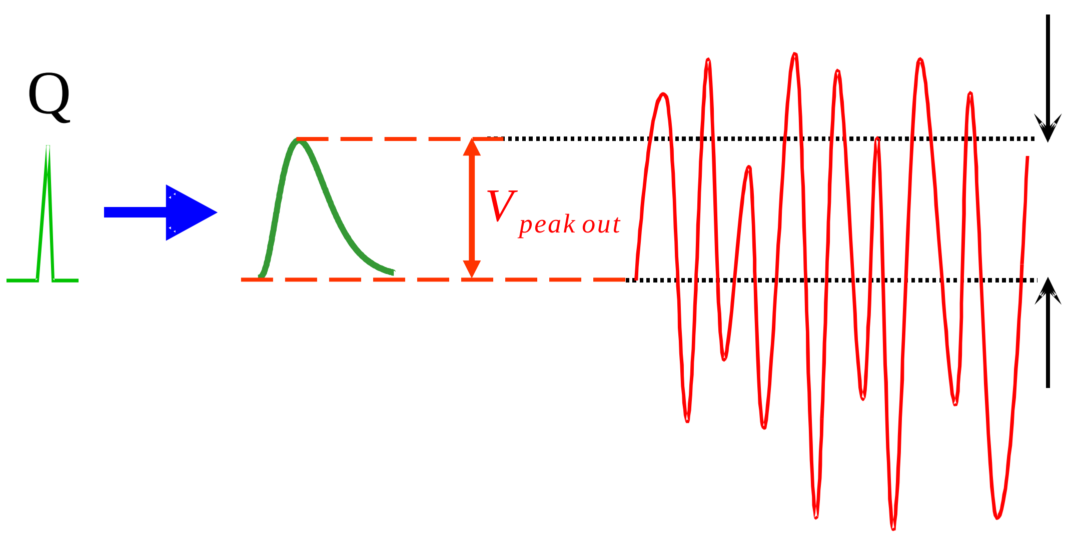
$$\sigma^2_{N_{\mathrm{meas}}} = \sigma^2_{N_{\mathrm{c}}} + \sigma^2_{N_{\mathrm{elect}}}$$

Fano factor F, in silicon $F \approx 0.12$

The concept of Equivalent Noise Charge: ENC



$$\sigma^2_{N_{meas}} = \sigma^2_{N_c} + \sigma^2_{N_{elect}} \longrightarrow \boxed{ENC^2 = q^2\sigma^2_{N_{elect}}}$$

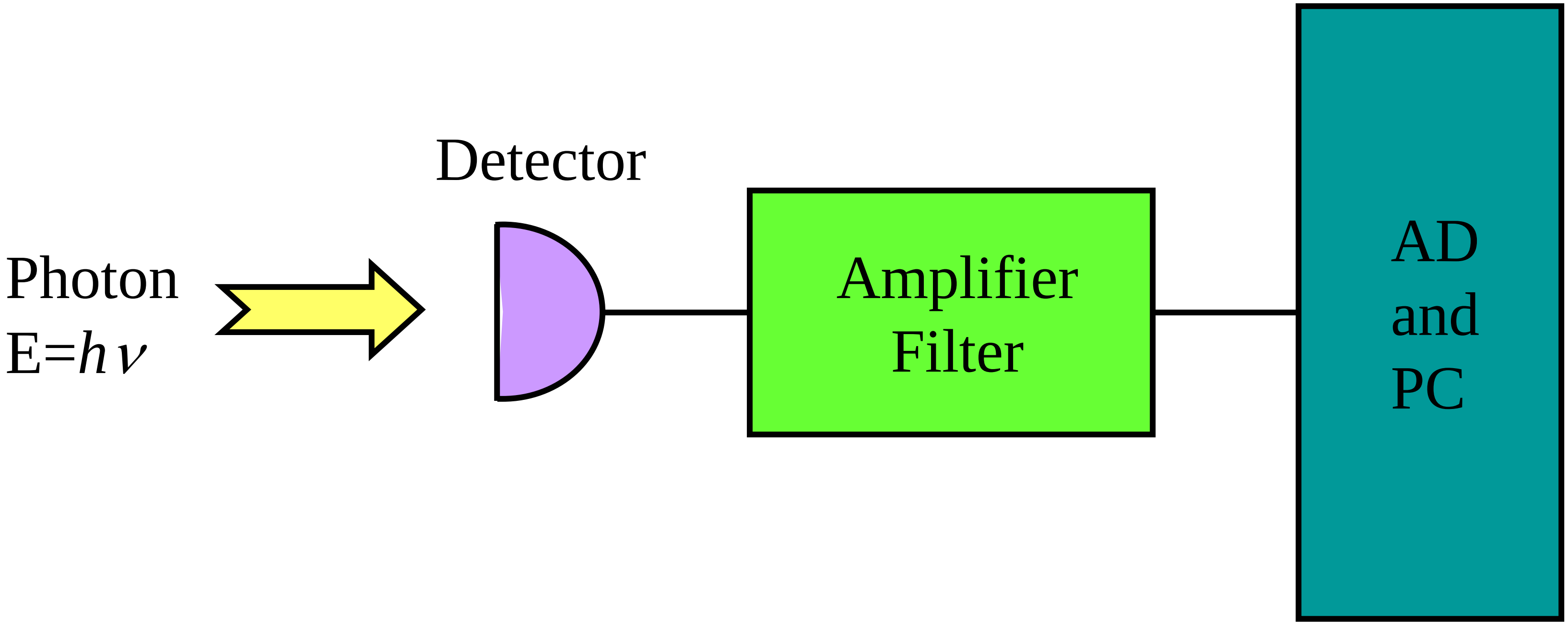


$$Q = ENC \quad \Leftrightarrow \quad \frac{S}{N} = 1$$

A lower ENC means a better resolution of the detection system

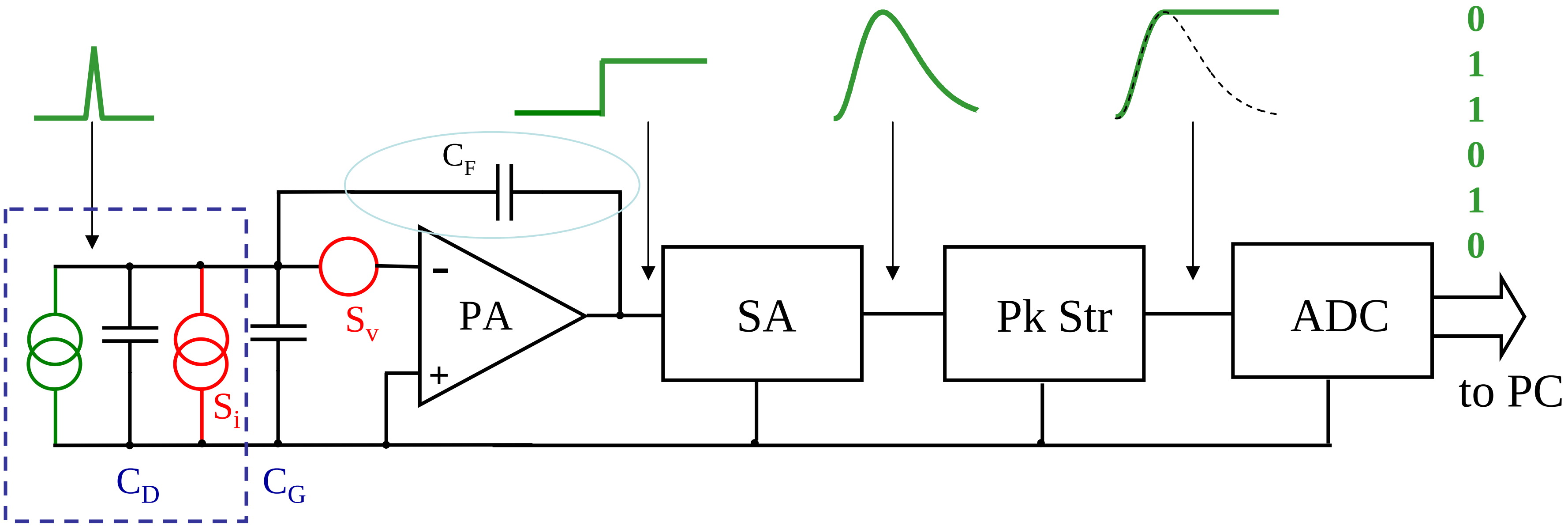
Signal and noise in the detection system

Block diagram of a typical detection system for X-ray spectroscopy



Our goal: to reach S/N as good as possible

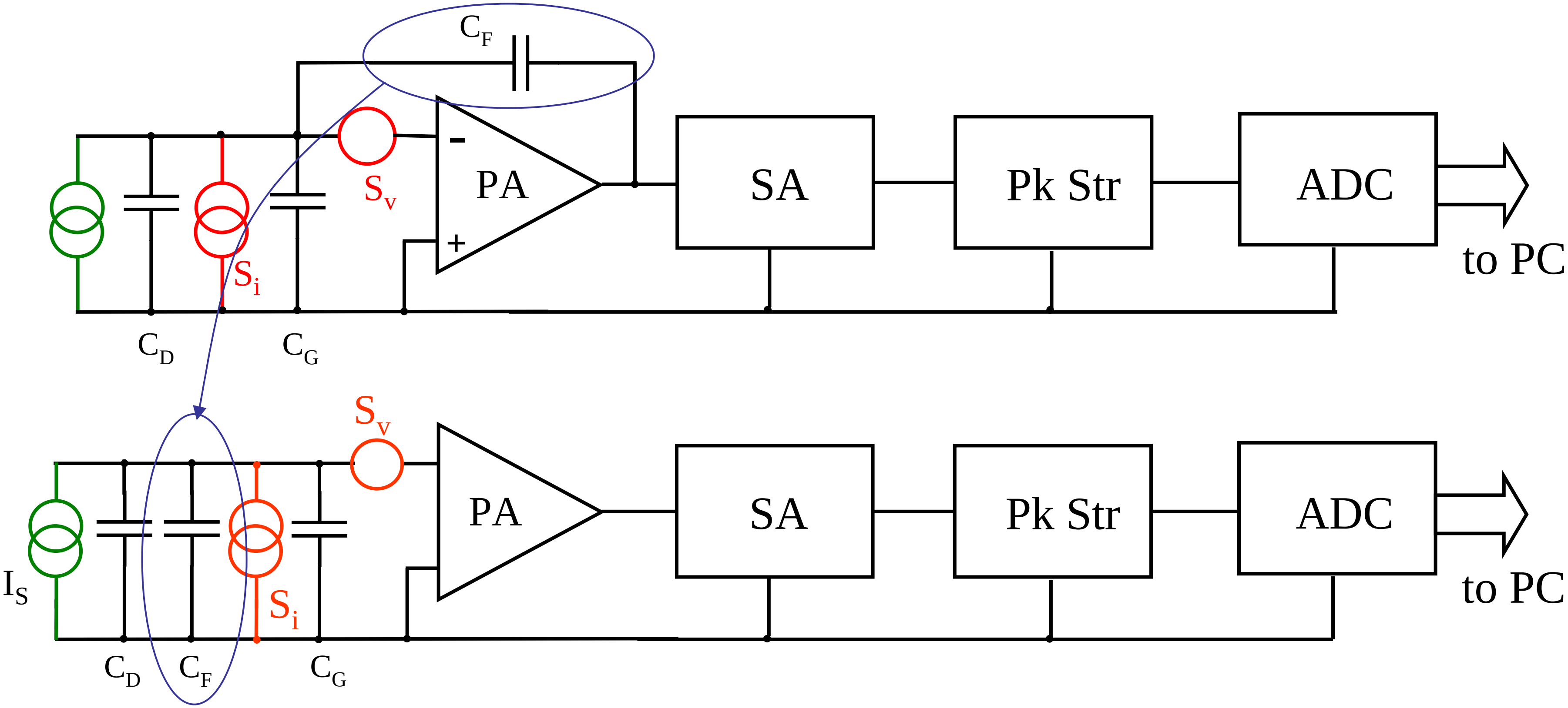
Block diagram of a typical detection system for X-ray spectroscopy: signal and noise



Detector

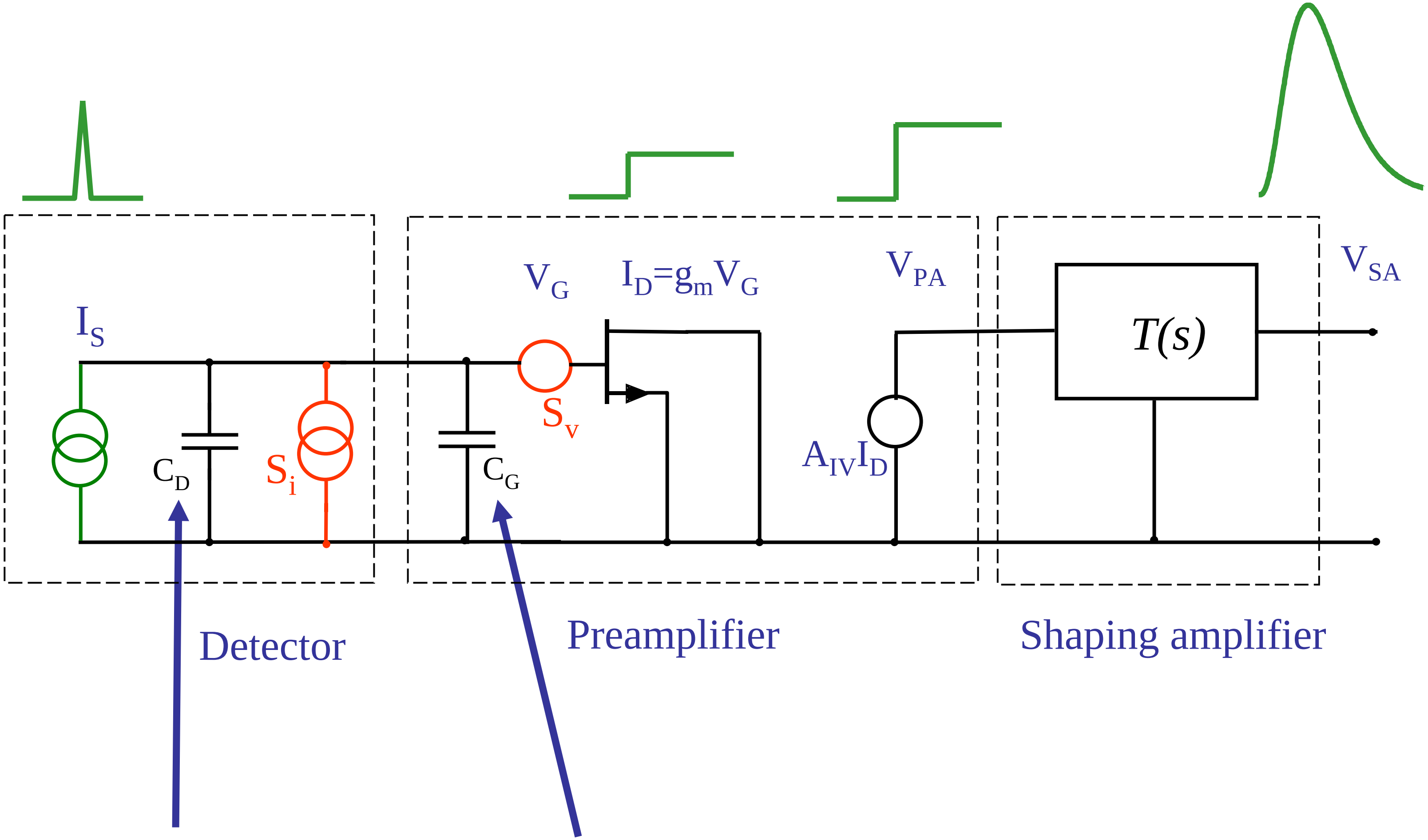
C_D comprises also
the parasitic capacitances
seen by the amplifier input

For what concerns S/N, charge-preamplifier and voltage preamplifier are equivalent



We will use voltage preamplifier because it is simpler to analyze

Block diagram: relevant elements



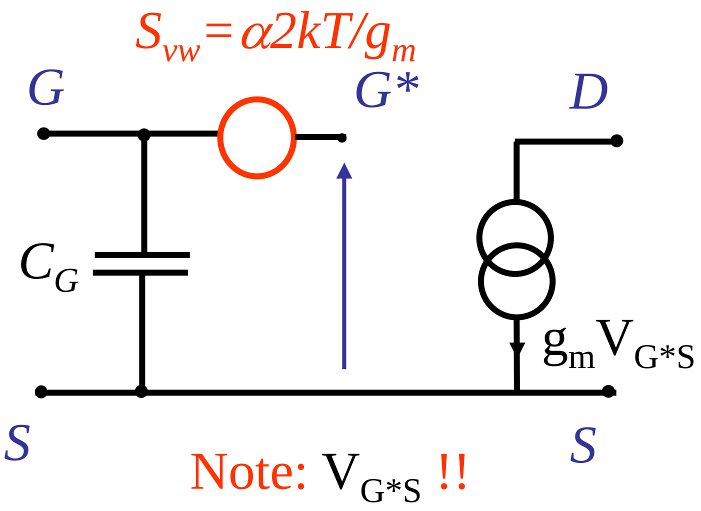
From now on C_D comprises also the feedback capacitance C_F:

$C_D=C_{det}+C_{parasitic}+C_{feedback}$

C_G is the input capacitance of the FET

White series noise of the FET

Thermal noise of the FET channel



Note: V_{G^*S} !!

$$\boxed{S_{vw} = \alpha \frac{2kT}{g_m} = \alpha \frac{2kT}{C_G}\frac{1}{\omega_T}}$$

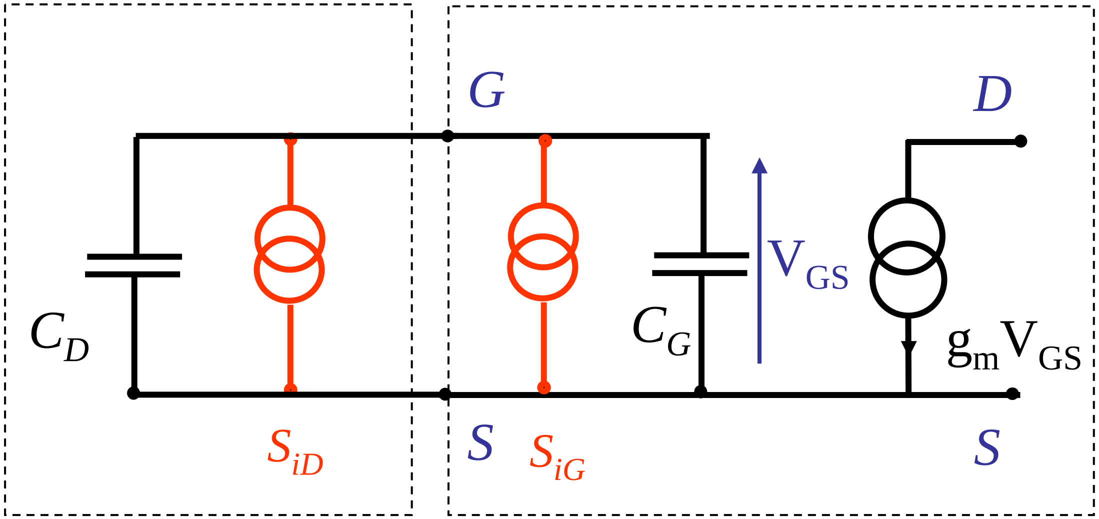
$\alpha \approx 2/3$

Note: bilateral noise spectra

White parallel noise of Detector and FET

Detector shot noise $S_{iD} = qI_D$

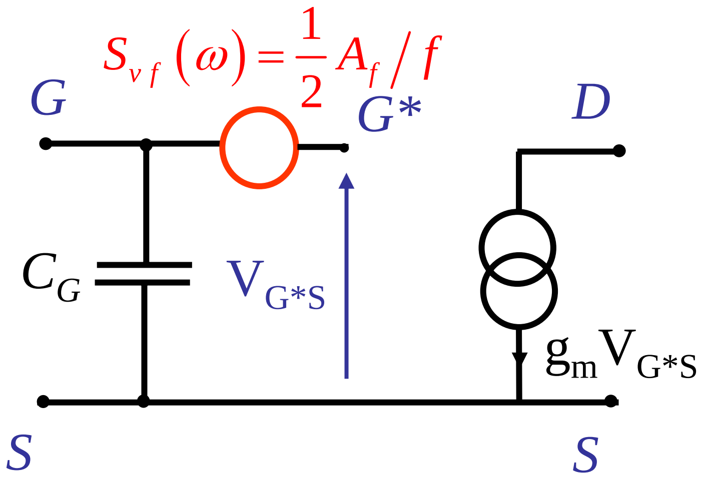
FET gate shot noise $S_{iG} = qI_G$



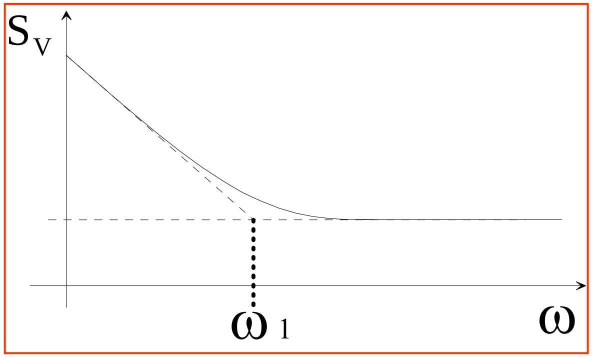
$$\boxed{S_{iw} = S_{iD} + S_{iG} = q(I_D + I_G) = qI_L}$$

1/f series noise – “flicker noise”

Mainly due to charge trapping and de-trapping
in the FET channel

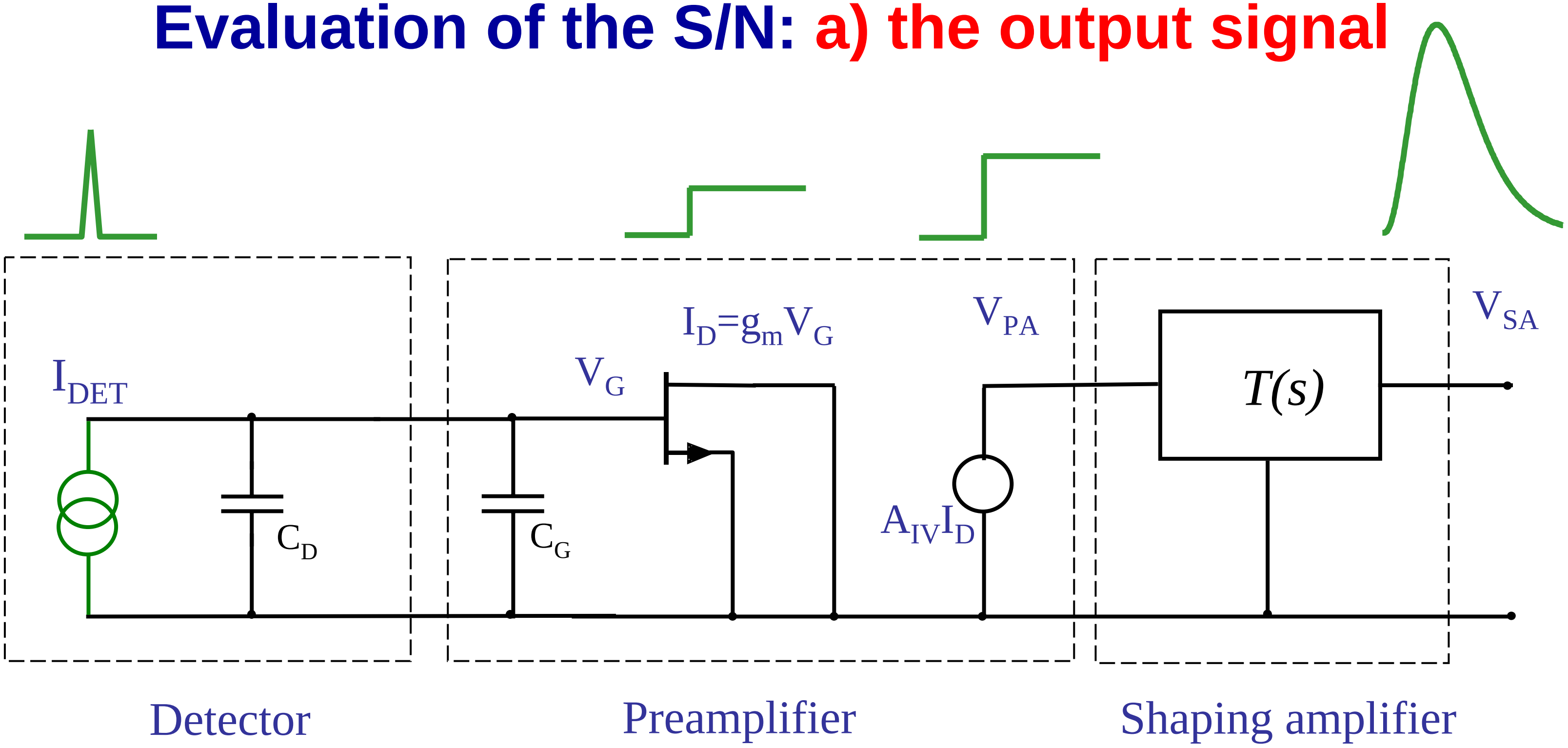


$$S_{vf}(\omega)=\frac{1}{2}\frac{A_f}{|f|}=\frac{\pi A_f}{|\omega|}=\alpha\frac{2kT}{C_G}\frac{\omega_1}{\omega_T}\frac{1}{|\omega|}$$



$$\frac{\omega_1}{\omega_T}=\frac{\pi A_f C_G}{\alpha 2kT}=\frac{\pi}{\alpha 2kT}H_f \approx \text{ constant for a given technology}$$

Evaluation of the S/N: a) the output signal



$$\boxed{I_{DET}(s)=Q}$$

$$V_G(s)=\frac{Q}{C_T}\frac{1}{s}$$

$$I_{DRAIN}(s)=\frac{Q}{C_T}g_m\frac{1}{s}$$

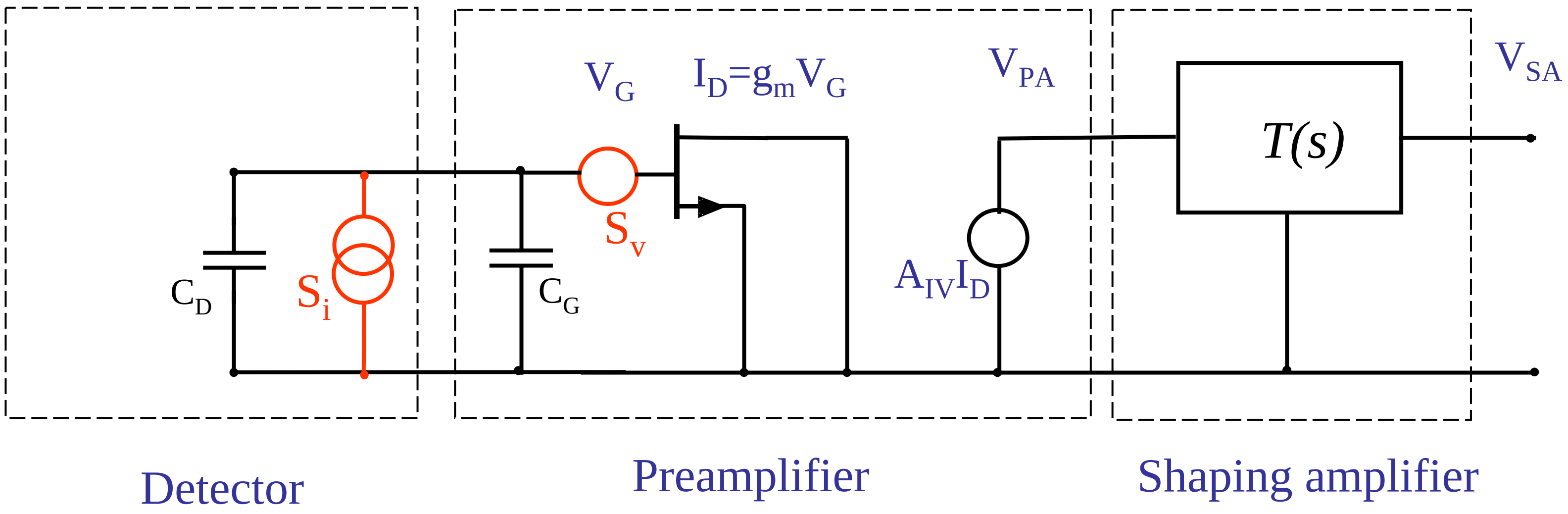
$$V_{PAout}(s)=\frac{Q}{C_T}g_mA_{IV}\frac{1}{s}$$

$$\boxed{V_{SAout}(s)=Q\frac{g_mA_{IV}}{C_T}\frac{1}{s}T(s)}$$

$$C_T=C_D+C_G$$

$$\left(\frac{S}{N}\right)^2=\frac{V^2_{SAout\ peak}}{\left\langle v^2_{no}\right\rangle}$$

Evaluation of the S/N: b) the output noise



$$S_{no}(\omega)=\left(S_{vw}+S_{vf}(\omega)+\frac{S_{iw}}{\omega^2C_T^2}\right)g_m^2A_{IV}^2\left|T(j\omega)\right|^2$$

$$\left\langle v_{no}^2\right\rangle=\frac{1}{2\pi}\int_{-\infty}^{+\infty}S_{no}(\omega)\,d\omega$$

$$\left(\frac{S}{N}\right)^2=\frac{V_{peak\,out}^2}{\left\langle v_{no}^2\right\rangle}$$

Evaluation of the S/N

$$\frac{S^2}{N^2}=\frac{V_{peak\,out}^2}{\left\langle v_{no}^2\right\rangle}=\frac{A_{IV}^2\,\dfrac{Q^2}{C_T^2}\,g_m^2 Max^2\left\{L^{-1}\left[\dfrac{1}{s}T(s)\right]\right\}}{\dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{+\infty}\left(S_{vw}+S_{vf}(\omega)+\dfrac{S_{iw}}{\omega^2 C_T^2}\right)g_m^2 A_{IV}^2\left|T\left(j\omega\right)\right|^2 d\omega}$$

The ENC does not depend on the gain of the shaper ⇒ $Max^2\left(L^{-1}\left[\frac{1}{s}T(s)\right]\right)=1$

The Equivalent Noise Charge

Evaluation of the ENC

$$Q = ENC \quad \Leftrightarrow \quad \frac{S}{N} = 1$$

$$ENC^2 = \frac{1}{2\pi} C_T^2 \int_{-\infty}^{+\infty} \left(S_{vw} + S_{vf}(\omega) + \frac{S_{iw}}{\omega^2 C_T^2} \right) \left| T(j\omega) \right|^2 d\omega$$

$$ENC^2 = C_T^2 S_{vw} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left| T(j\omega) \right|^2 d\omega + C_T^2 \pi A_f \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{|\omega|} \left| T(j\omega) \right|^2 d\omega + S_{iw} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{\omega^2} \left| T(j\omega) \right|^2 d\omega$$

Series noise contribution

1/f noise contribution

Parallel noise contribution

The “shape factors”

The angular frequency ω can be normalised to a characteristic frequency $\omega_c=1/\tau$, where τ is a characteristic time which represents the width of the output pulse (for instance the peaking time, or the time width at half height, or a characteristic time constant of the filter). The characteristic time τ is also called ‘shaping time’ of the filter.

$$\boxed{x = \frac{\omega}{\omega_c} = \omega\tau}$$

$$\boxed{ENC^2 = C_T^2 S_{vw} \frac{1}{\tau} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left|T(jx)\right|^2 dx + C_T^2 \pi A_f \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{|x|} \left|T(jx)\right|^2 dx + S_{iw} \tau \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{\omega^2} \left|T(jx)\right|^2 dx}$$

Series noise
contribution

1/f noise
contribution

Parallel noise
contribution

$$\boxed{\begin{aligned} A_1 &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left|T(jx)\right|^2 dx \\ A_2 &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{|x|}\left|T(jx)\right|^2 dx \\ A_3 &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{x^2}\left|T(jx)\right|^2 dx \end{aligned}}$$

The three factors A_1, A_2 and A_3 depend on the SHAPE of the output pulse and on the choice of the characteristic time τ of the output pulse used to normalise the angular frequency ω.

The factors A_1, A_2 and A_3 do not depend on the particular value of the shaping time τ.

$$\tau'' = k\tau' \longrightarrow \begin{aligned} A_1(\tau'') &= k A_1(\tau') \\ A_2(\tau'') &= A_2(\tau') \\ A_3(\tau'') &= \frac{1}{k} A_3(\tau') \end{aligned}$$

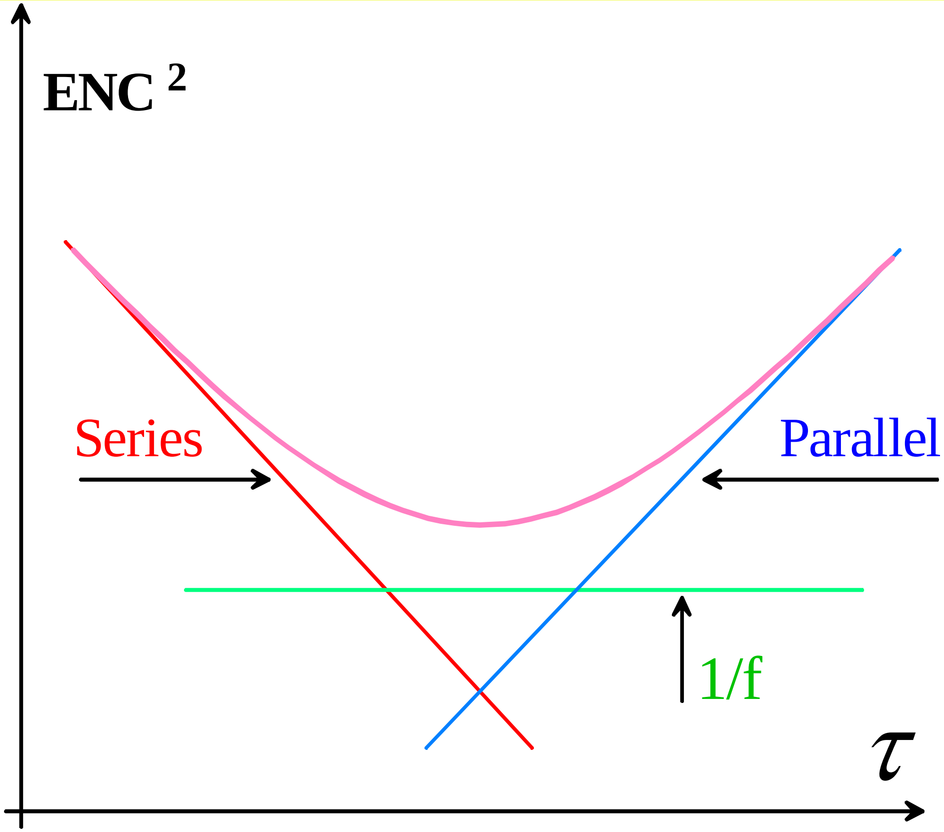
$$ENC^2 = C_T^2 S_{vw} \frac{1}{\tau} A_1 + C_T^2 \pi A_f A_2 + S_{iw} \tau A_3$$

SHAPE FACTORS						
		A_1	A_2	A_3	$\sqrt{A_1 A_3}$	$\sqrt{A_1 / A_3}$
Infinite cusp	τ	1.00	0.64	1.00	1.00	1.00
Triangular	$T_{base}/2$	2.00	0.88	0.67	1.16	1.73
Gaussian	σ	0.89	1.00	1.77	1.26	0.71
CR-RC	RC	1.85	1.18	1.85	1.85	1.00
CR-RC^4	RC	0.51	1.04	3.58	1.35	0.38
CR-RC^4	τ_{peak}	3.06	1.04	0.60	1.35	2.26
Semigaussian 7 poles	σ	0.92	1.03	1.83	1.30	0.71
Semigaussian 7 poles	τ_{peak}	2.70	1.03	0.62	1.30	2.08
Trapezoidal (T_f=0.5xT_r)	T_r	2.00	1.18	1.16	1.52	1.31
Trapezoidal (T_f=T_r)	T_r	2.00	1.38	1.67	1.83	1.09
Trapezoidal (T_f=2xT_r)	T_r	2.00	1.64	2.67	2.31	0.87

The dependence on τ of the 3 contributions to the ENC

$$ENC^2 = C_T^2 S_{vw} \frac{1}{\tau} A_1 + C_T^2 \pi A_f A_2 + S_{iw} \tau A_3$$

$$ENC^2 = ENC^2_{series} + ENC^2_{1/f} + ENC^2_{parallel}$$



The dependence on transistor and detector parameters of the ENC

$$ENC^2 = C_T^2 S_{vw}\frac{1}{\tau}A_1 + C_T^2 \pi A_f A_2 + S_{iw}\tau A_3 \longleftarrow \begin{cases} S_{vw} = \alpha\dfrac{2kT}{g_m} = \alpha\dfrac{2kT}{C_G}\dfrac{1}{\omega_T} \\ S_{iw} = q(I_D + I_G + I_{Req}) = qI_L \\ S_{vf}(\omega) = \dfrac{\pi A_f}{|\omega|} = \alpha\dfrac{2kT}{C_G}\dfrac{\omega_1}{\omega_T}\dfrac{1}{|\omega|} \end{cases}$$

$$\Big\downarrow$$

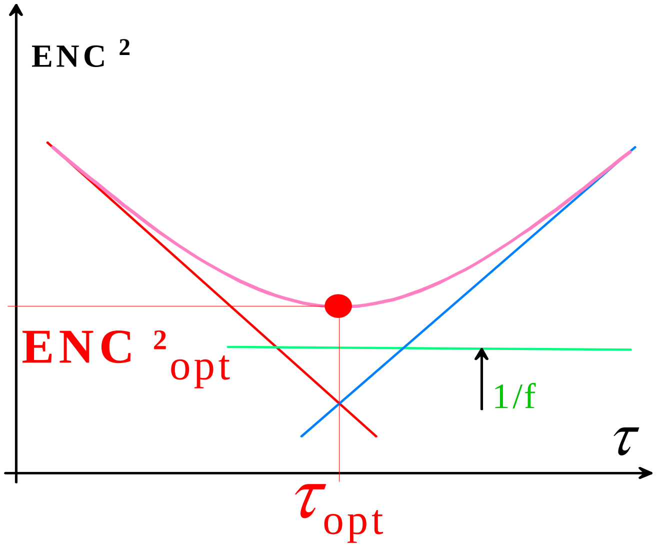
$$\boxed{ENC^2} = \underbrace{\boxed{A_1 C_D\left(\sqrt{\frac{C_D}{C_G}} + \sqrt{\frac{C_G}{C_D}}\right)^2 \alpha\frac{2kT}{\omega_T}\frac{1}{\tau}}}_{\text{Series noise}} + \underbrace{\boxed{A_2 C_D\left(\sqrt{\frac{C_D}{C_G}} + \sqrt{\frac{C_G}{C_D}}\right)^2 \alpha\frac{2kT}{\omega_T}\omega_1}}_{\text{1/f noise}} + \underbrace{\boxed{A_3 q I_T \tau}}_{\text{Parallel noise}}$$

How to optimise the ENC

How to optimise the ENC (1)

Choose the optimum shaping time

$$ENC^2_{series} = ENC^2_{parallel}$$



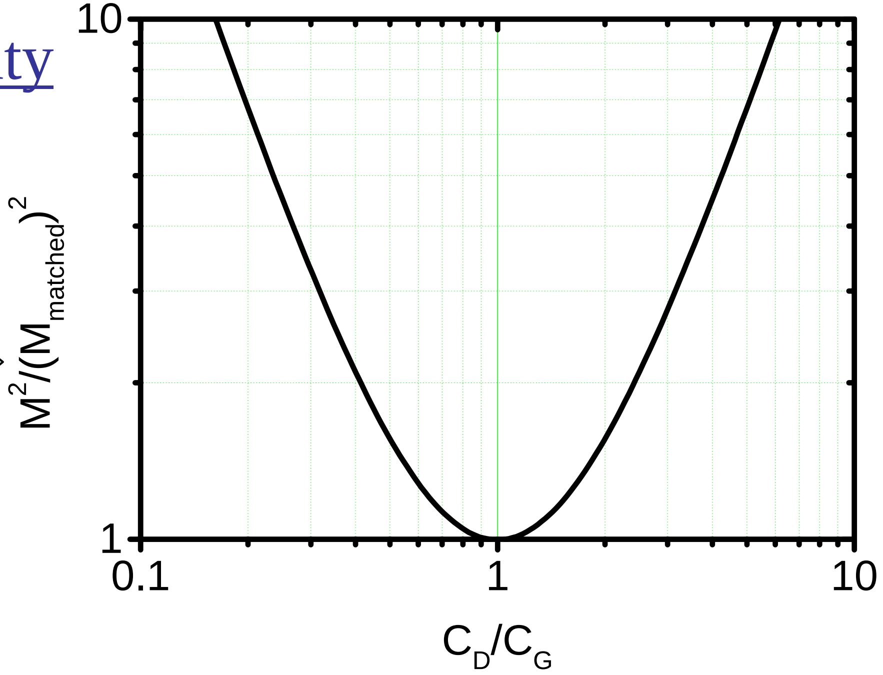
$$\tau_{opt} = \left(C_D + C_G\right)\sqrt{\frac{S_{Vw}}{S_{Iw}}}\sqrt{\frac{A_1}{A_3}} = \sqrt{C_D}\left(\sqrt{\frac{C_D}{C_G}} + \sqrt{\frac{C_G}{C_D}}\right)\sqrt{\alpha\frac{2kT}{\omega_T}\frac{1}{qI_L}}\sqrt{\frac{A_1}{A_3}}$$

How to optimise the ENC (2)

Match the Detector capacitance with the Gate capacitance

* For constant FET current density

$$M^2 = \left(\sqrt{\frac{C_D}{C_G}} + \sqrt{\frac{C_G}{C_D}} \right)^2$$

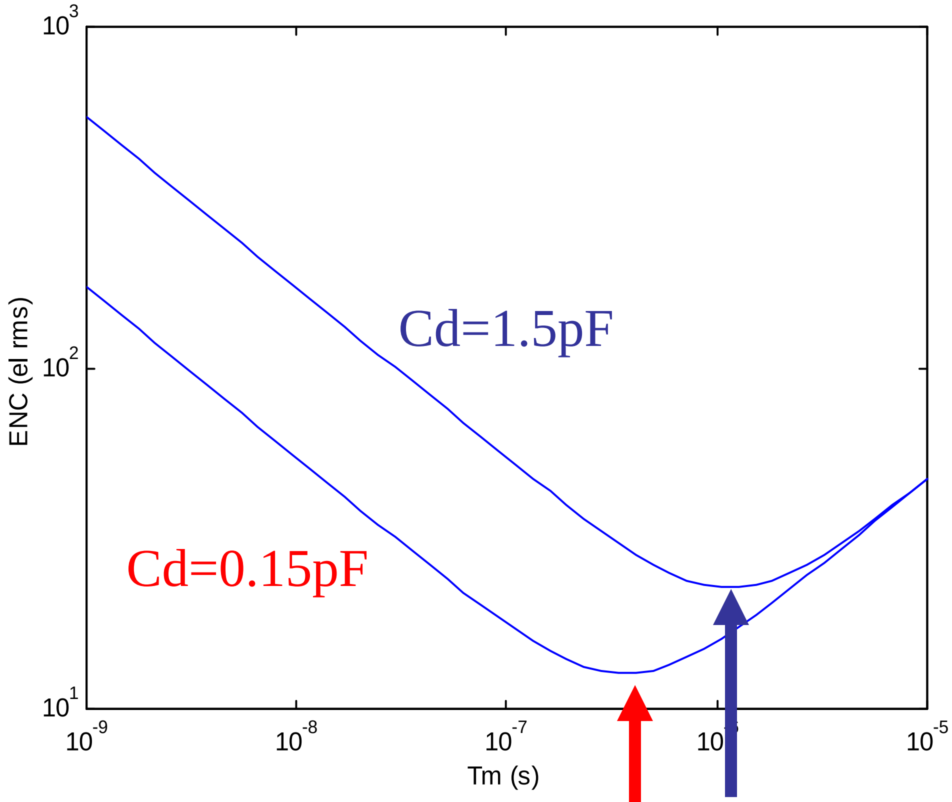


Series white and series 1/f noise contributions are minimised

$$ENC^2 = \boxed{A_1 C_D \left(\sqrt{\frac{C_D}{C_G}} + \sqrt{\frac{C_G}{C_D}} \right)^2 \alpha \frac{2kT}{\omega_T} \frac{1}{\tau}} + \boxed{A_2 C_D \left(\sqrt{\frac{C_D}{C_G}} + \sqrt{\frac{C_G}{C_D}} \right)^2 \alpha \frac{2kT}{\omega_T} \omega_1} + A_3 q I_T \tau$$

How to optimise the ENC (3)

Reduce the detector capacitance (in matched conditions)



$$ENC_{opt} \approx \sqrt{\frac{I_{leak} C_D}{\omega_T}}$$

$$\tau_{opt} = \sqrt{\frac{C_D}{I_{leak}\omega_T}}$$

* With 1/f noise negligible

ENC_{opt} = 13 e- rms

τ_{opt}= 0.4 µs

ENC_{opt} = 23 e- rms

τ_{opt}= 1.3 µs

Series white and series 1/f noise contributions are minimised

How to optimise the ENC (4)

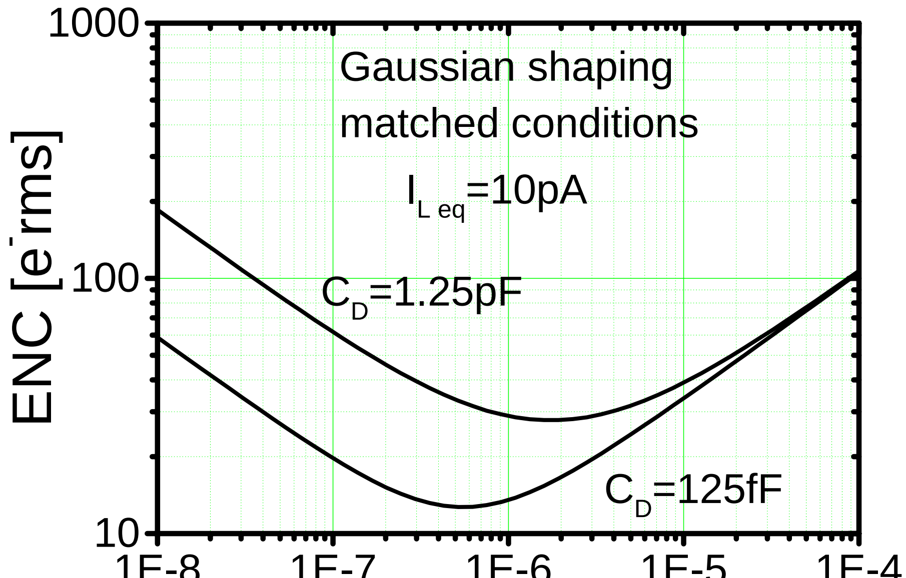
Reduce the parallel noise sources

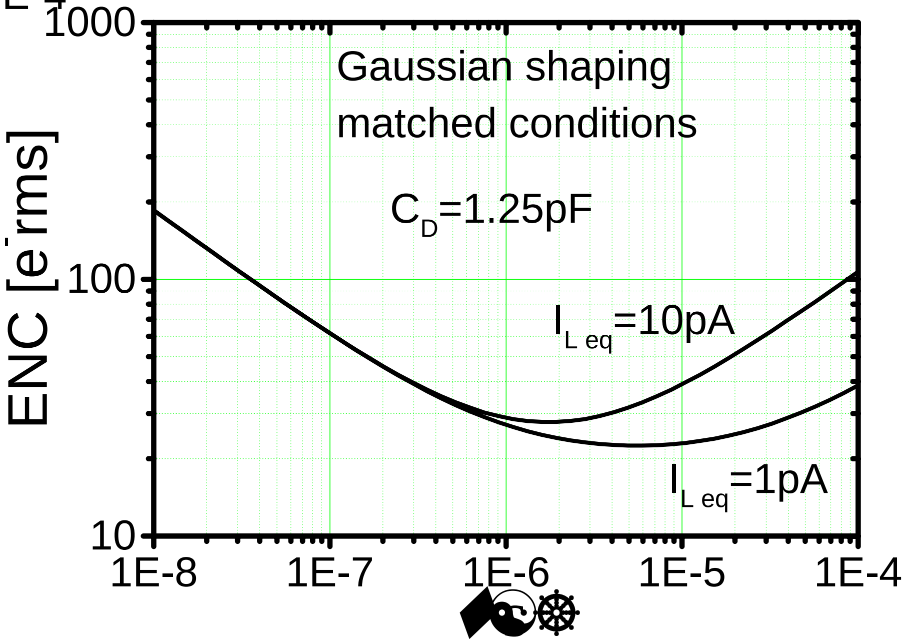
- Reduce the detector leakage current by
  - cooling the detector
  - reducing the detector active volume
  - improving the detector technology
- Increase the value of the resistors connected to the FET input (bias or feedback resistors)

Parallel white noise contribution is minimised

$$ENC^2 = A_1 C_D \left(\sqrt{\frac{C_D}{C_G}} + \sqrt{\frac{C_G}{C_D}} \right)^2 \alpha \frac{2kT}{\omega_T} \frac{1}{\tau} + A_2 C_D \left(\sqrt{\frac{C_D}{C_G}} + \sqrt{\frac{C_G}{C_D}} \right)^2 \alpha \frac{2kT}{\omega_T} \omega_1 + \boxed{A_3 q I_T \tau}$$

C_D and I_L


Gaussian shaping
matched conditions
$I_{L\ eq}$=10pA
C_D=1.25pF
C_D=125fF
ENC [e⁻rms]
1000
100
10
1E-8
1E-7
1E-6
1E-5
1E-4


Gaussian shaping
matched conditions
C_D=1.25pF
$I_{L\ eq}$=10pA
$I_{L\ eq}$=1pA
ENC [e⁻rms]
1000
100
10
1E-8
1E-7
1E-6
1E-5
1E-4

How to optimise the ENC (5)

Choose the best transistor

$$ENC^2 = A_1 C_D \left(\sqrt{\frac{C_D}{C_G}} + \sqrt{\frac{C_G}{C_D}} \right)^2 \alpha \frac{2kT}{\omega_T} \frac{1}{\tau} + A_2 C_D \left(\sqrt{\frac{C_D}{C_G}} + \sqrt{\frac{C_G}{C_D}} \right)^2 2kT \alpha \frac{\omega_1}{\omega_T} + A_3 q(I_D + I_G)\tau$$

Bandwidth

1/f noise

Gate Leakage

$$\boxed{\frac{\omega_1}{\omega_T} = \frac{\pi A_f C_G}{\alpha 2kT} = \frac{\pi}{\alpha 2kT} H_f}$$

DEVICE	**H_f [J]**
JFET, n-channel, discrete	2×10^{-26}
JFET, n-channel, in CMOS process	10^{-25}
MOSFET, p channel, in CMOS process	6×10^{-25}
MOSFET, n channel, in CMOS process	2.5×10^{-23}
MESFET, GaAs, discrete	10^{-23}

How to optimise the ENC (6)

Perform the optimum signal processing

If only the white noise sources (series and parallel) are present, the best ENC can be obtained by using an ideal filtering amplifier which gives at its output an ‘infinite cusp’- shaped pulse

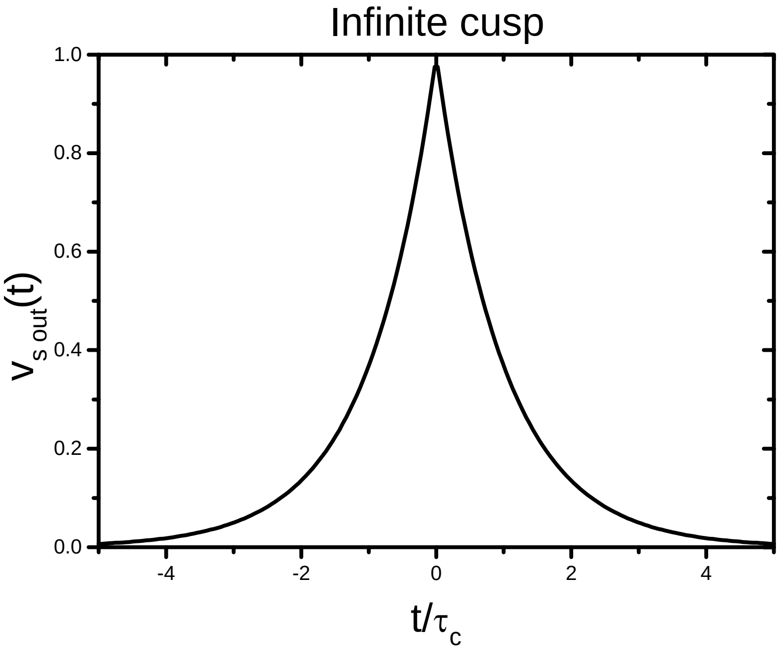
$$v_{s\,out}(t) = \exp\left(-\frac{|t|}{\tau}\right)$$

with a shaping time τ set equal to the ‘noise corner’ time constant τ_c.

$$\tau_c = (C_D + C_G)\sqrt{\frac{S_{Vw}}{S_{Iw}}}$$

Ideal filter (for white noise sources)

Optimum signal processing



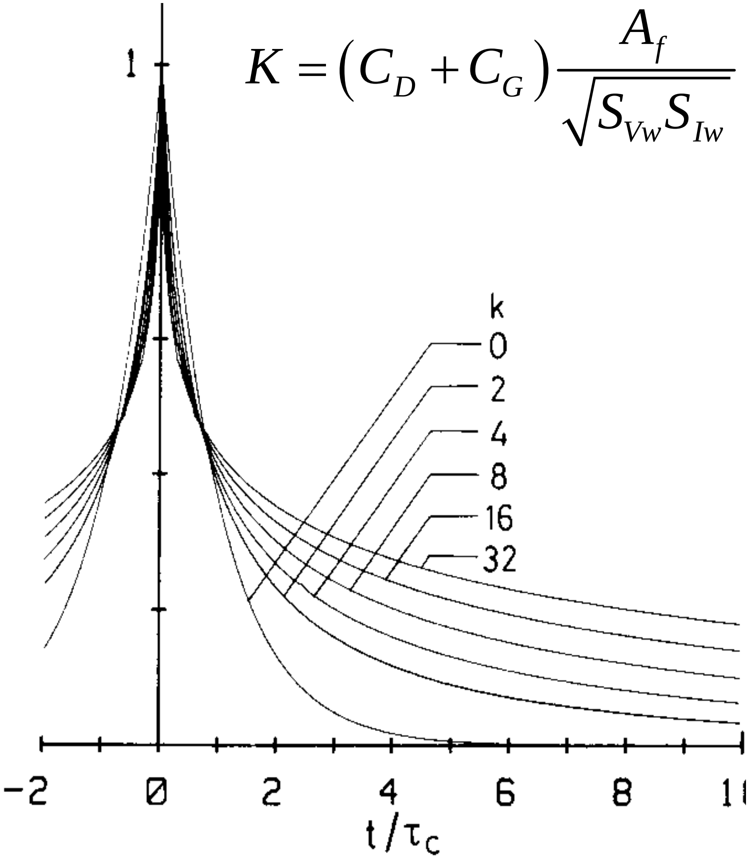
$$A_1 = 1 \quad A_2 = \frac{2}{\pi} \approx 0.64 \quad A_3 = 1$$

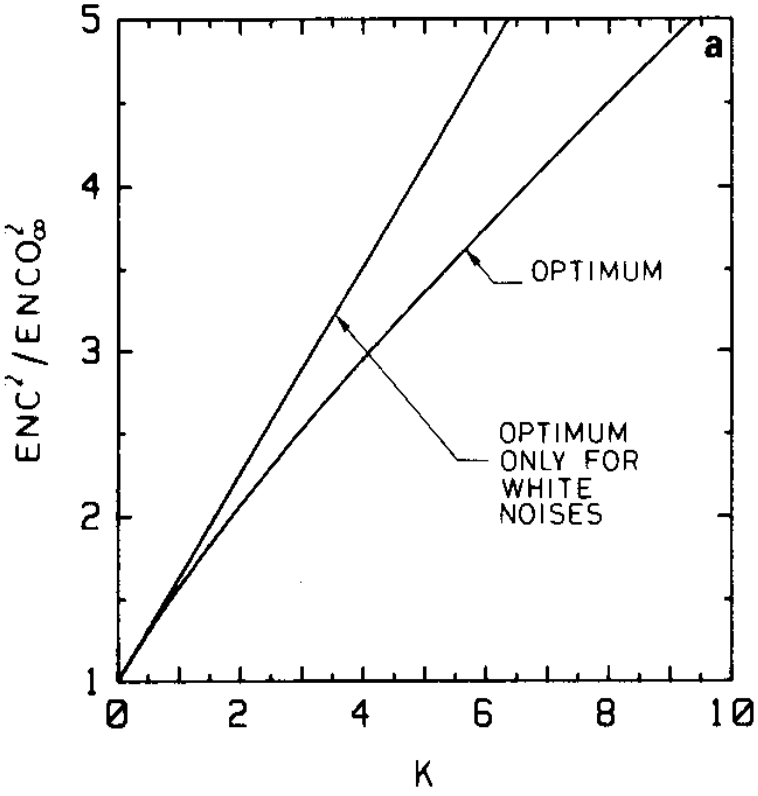
$$\boxed{ENC_\infty^2 = 2\left(C_D + C_G\right)\sqrt{S_{Vw} S_{Iw}}}$$

Ideal filter (In presence of white and 1/f noise)

Optimum signal processing

$$K = \left(C_D + C_G\right)\frac{A_f}{\sqrt{S_{Vw}S_{Iw}}}$$





Practical filters

$$ENC^2 = C_T^2 S_{vw} \frac{1}{\tau} A_1 + C_T^2 \pi A_f A_2 + S_{iw} \tau A_3$$

SHAPE FACTORS						
		A_1	A_2	A_3	$\sqrt{A_1 A_3}$	$\sqrt{A_1 / A_3}$
Infinite cusp	τ	1.00	0.64	1.00	1.00	1.00
Triangular	$T_{base}/2$	2.00	0.88	0.67	1.16	1.73
Gaussian	σ	0.89	1.00	1.77	1.26	0.71
CR-RC	RC	1.85	1.18	1.85	1.85	1.00
CR-RC^4	RC	0.51	1.04	3.58	1.35	0.38
CR-RC^4	τ_{peak}	3.06	1.04	0.60	1.35	2.26
Semigaussian 7 poles	σ	0.92	1.03	1.83	1.30	0.71
Semigaussian 7 poles	τ_{peak}	2.70	1.03	0.62	1.30	2.08
Trapezoidal (T_f=0.5xT_r)	T_r	2.00	1.18	1.16	1.52	1.31
Trapezoidal (T_f=T_r)	T_r	2.00	1.38	1.67	1.83	1.09
Trapezoidal (T_f=2xT_r)	T_r	2.00	1.64	2.67	2.31	0.87

Appendix:
Noise modelling

Noise modelling

A noise waveform can be represented as a random sequence of pulses:

$$x(t) = \sum_k a_k f(t - t_k)$$

The distribution of t_k is Poissonian*

The bilateral spectrum $S_x(\omega)$ *is given by:*

$$\boxed{\overline{S_x(\omega)} = \lambda \left| F(j\omega) \right|^2}$$

⇐ Carson’s theorem

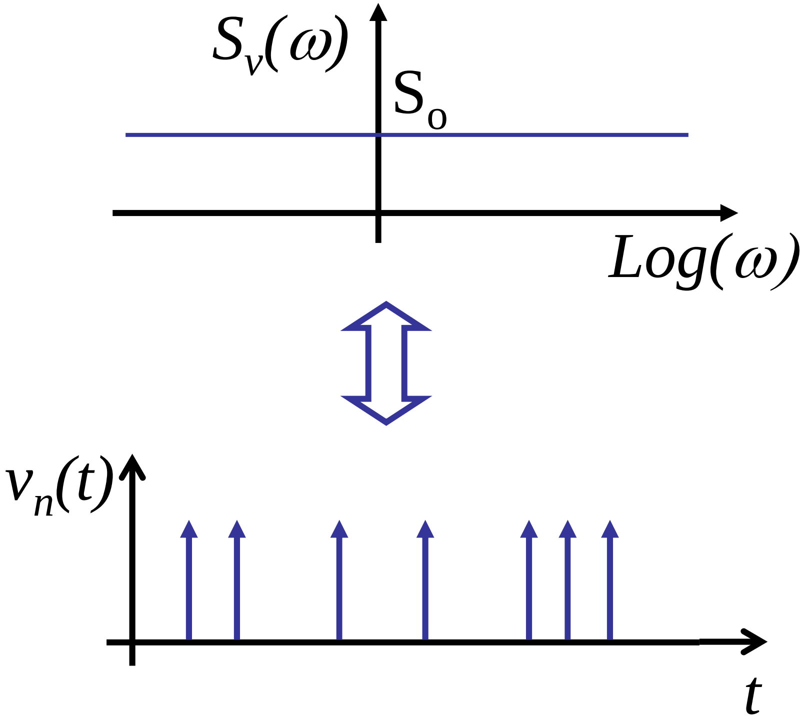
$\lambda =$ mean rate of pulses

$F(j\omega) =$ Fourier transform of the pulse shape f(t)

* Processes whose probability of occurrence is small and constant: $P_x = \dfrac{m^x}{x!}\exp(-m)$

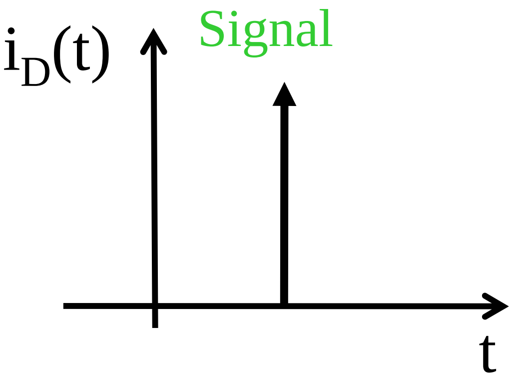
White noise modelling

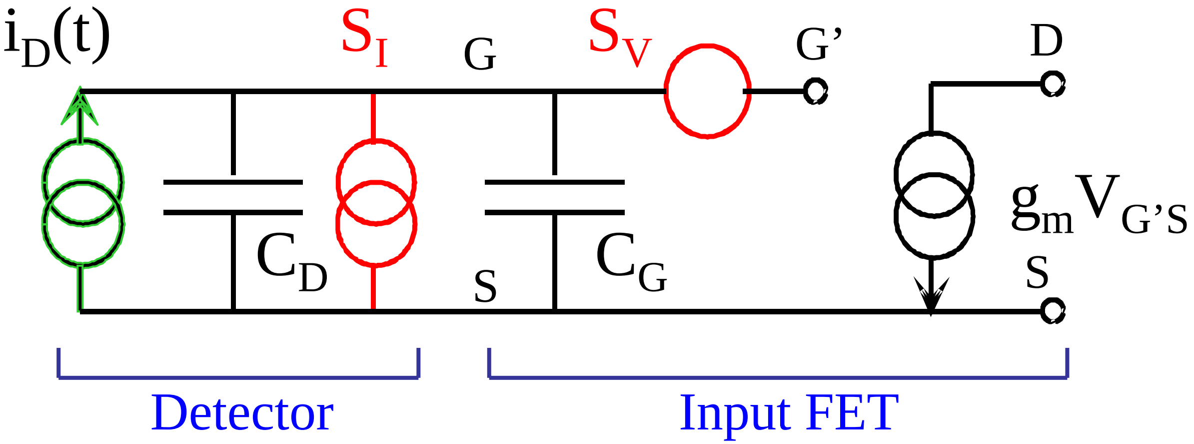
White noise with spectrum S_o can be modelled by
a random sequence of δ pulses
of unit area and average rate $\lambda = S_o$

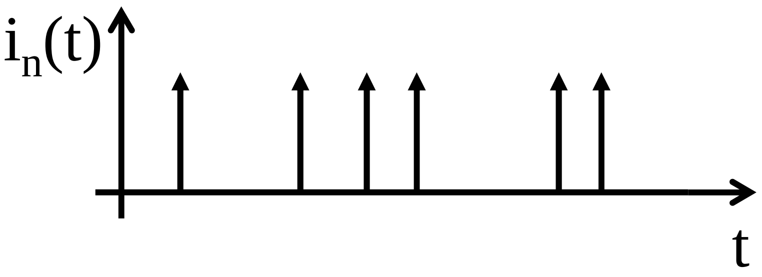


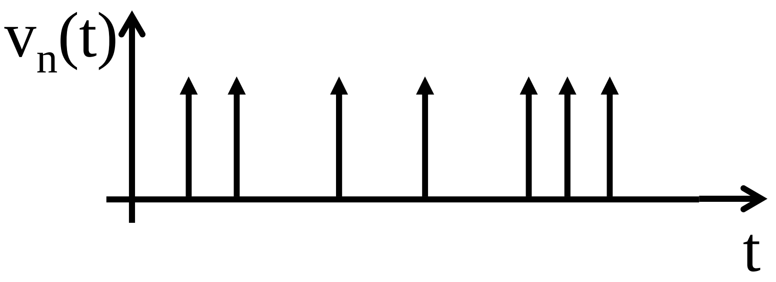
White noise modelling

Small signal model of the front-end.







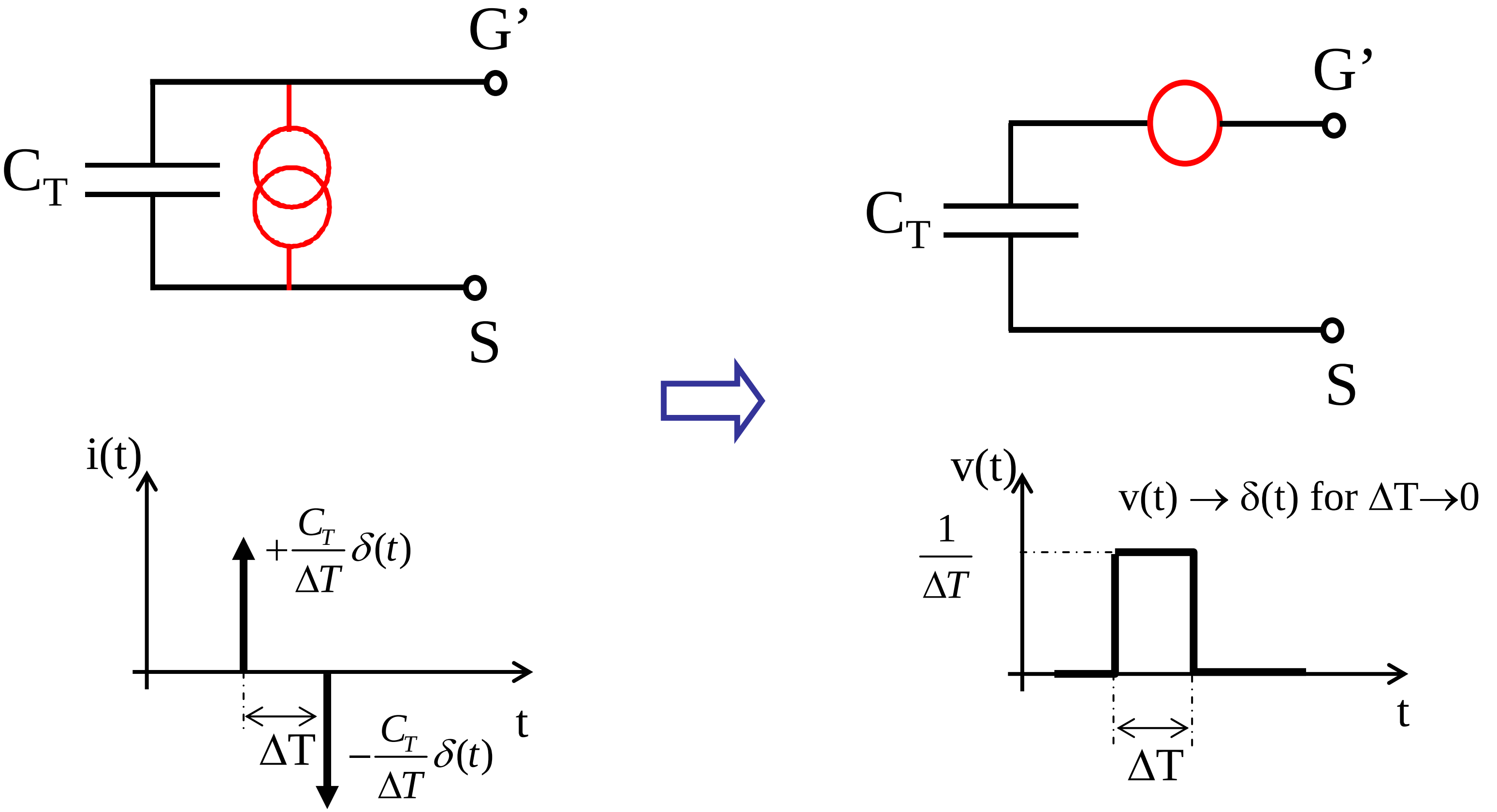


Parallel white noise can be modeled as a random sequence of current δ-pulses of unit area and average rate $\lambda_p = S_I$

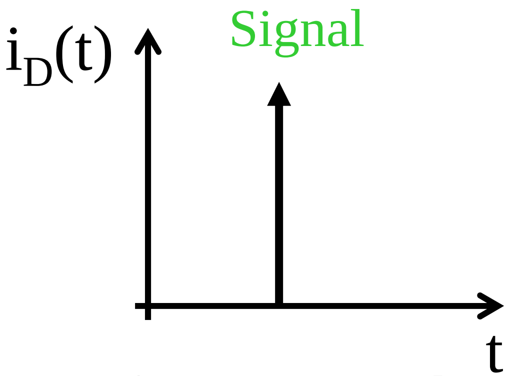
Series white noise can be modeled as a random sequence of voltage δ-pulses of unit area and average rate $\lambda_s = S_V$

White noise modelling

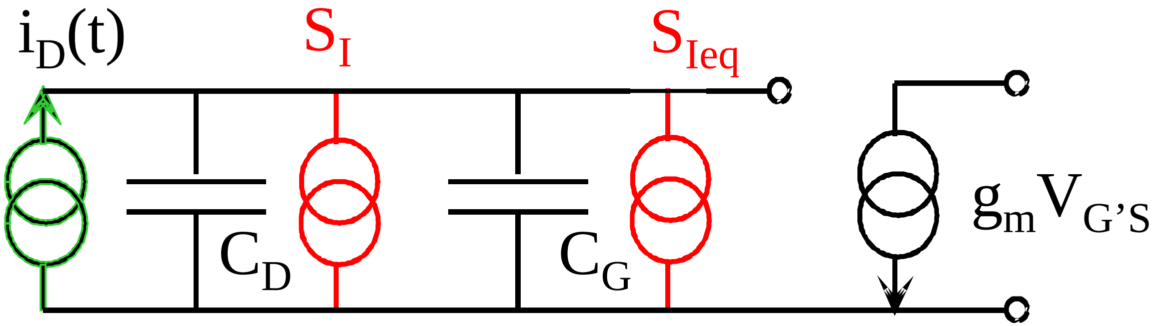
In order to better compare the noise with the current signal it is useful to transform the voltage noise generator in a current noise generator which gives the same noise signal at the input of the JFET



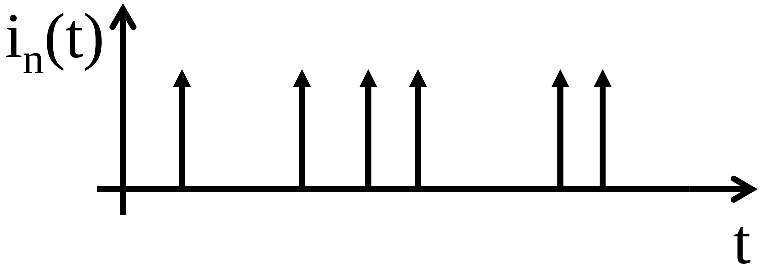
White noise modelling



δ(t) current pulse

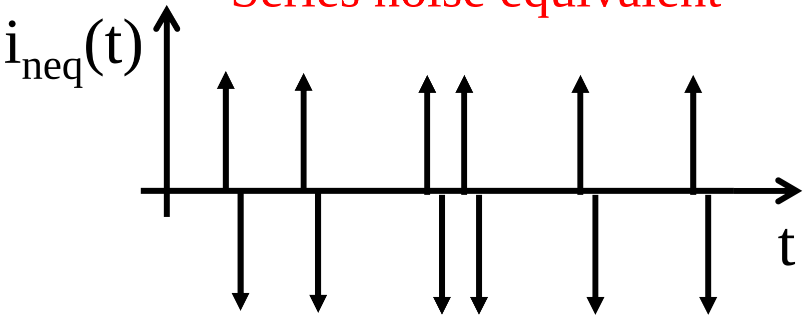


Parallel noise



δ(t) current pulses

Series noise equivalent



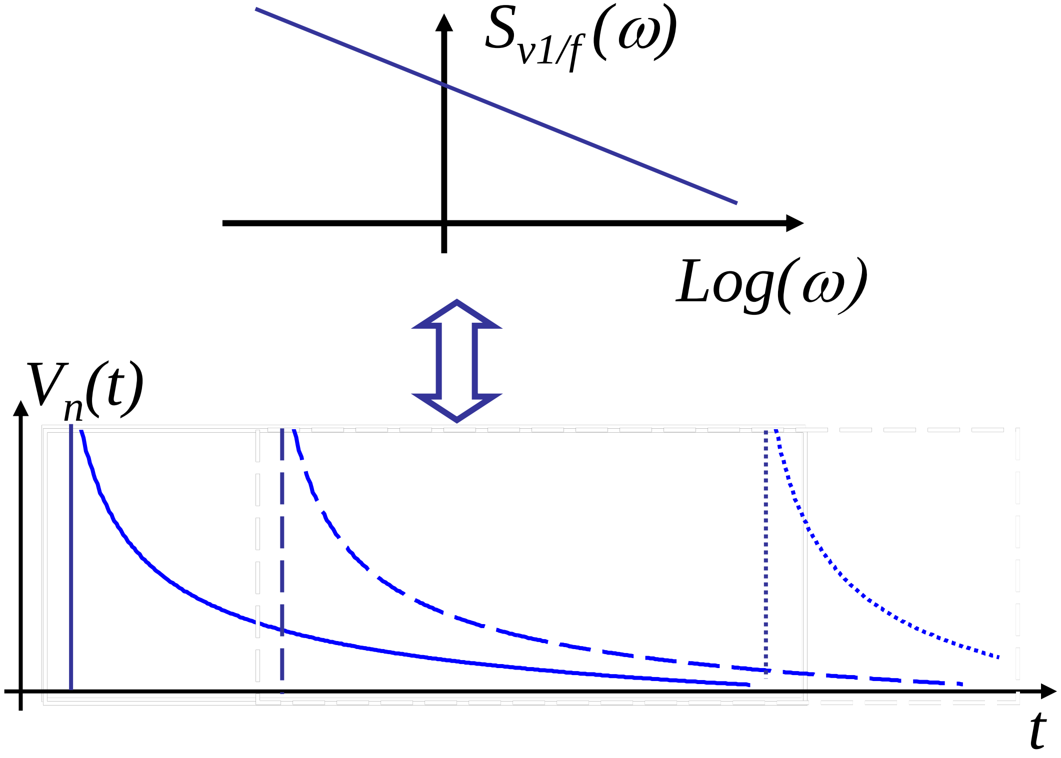
Doublets of δ(t) current pulses

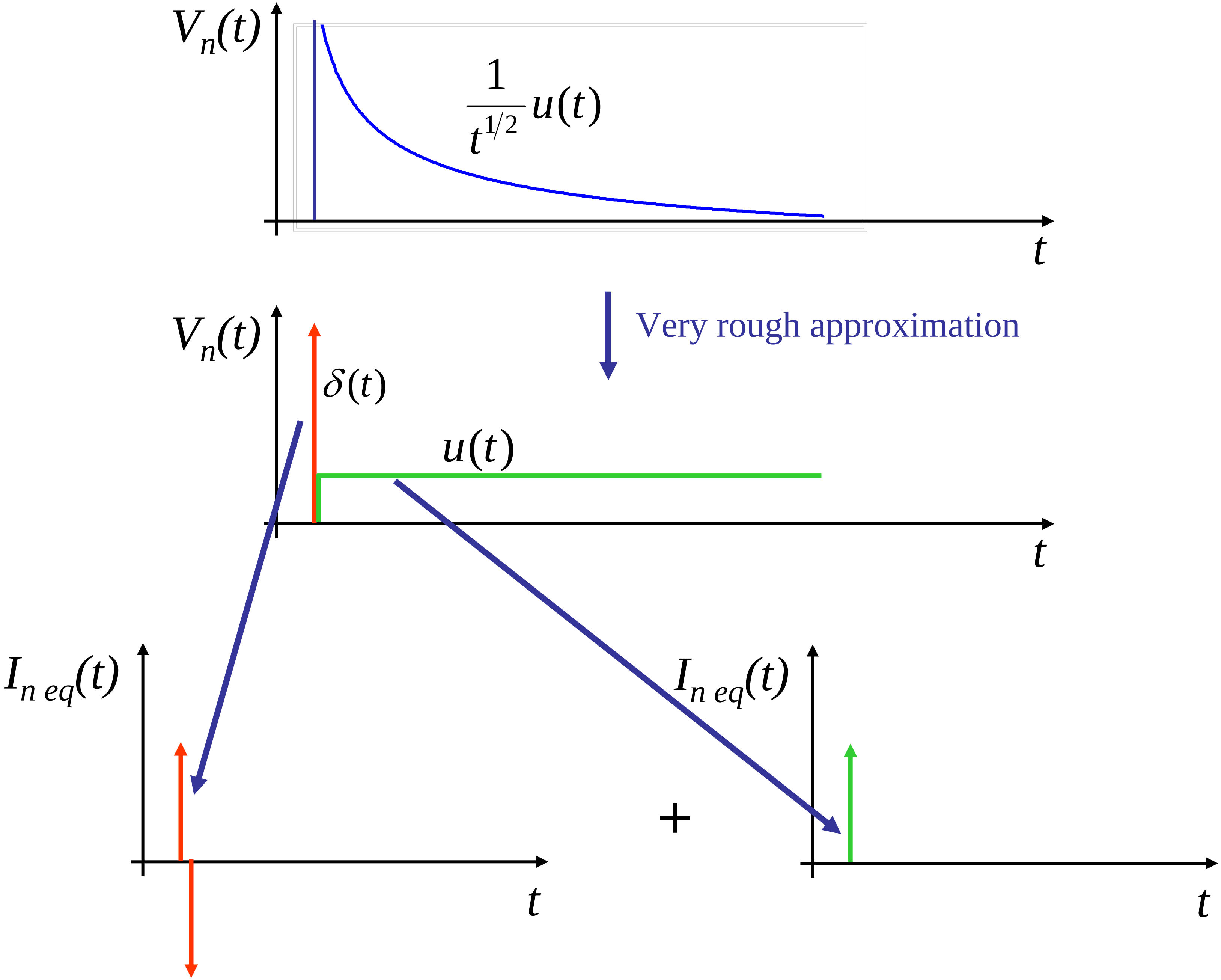
1/f noise modelling

The 1/f noise spectrum $S_{v\ f} = \frac{\pi A_f}{\omega}$

can be modelled with a random sequence of pulses $\varphi(t) = \frac{1}{t^{1/2}} u(t)$

occurring at a rate $r = A_f$

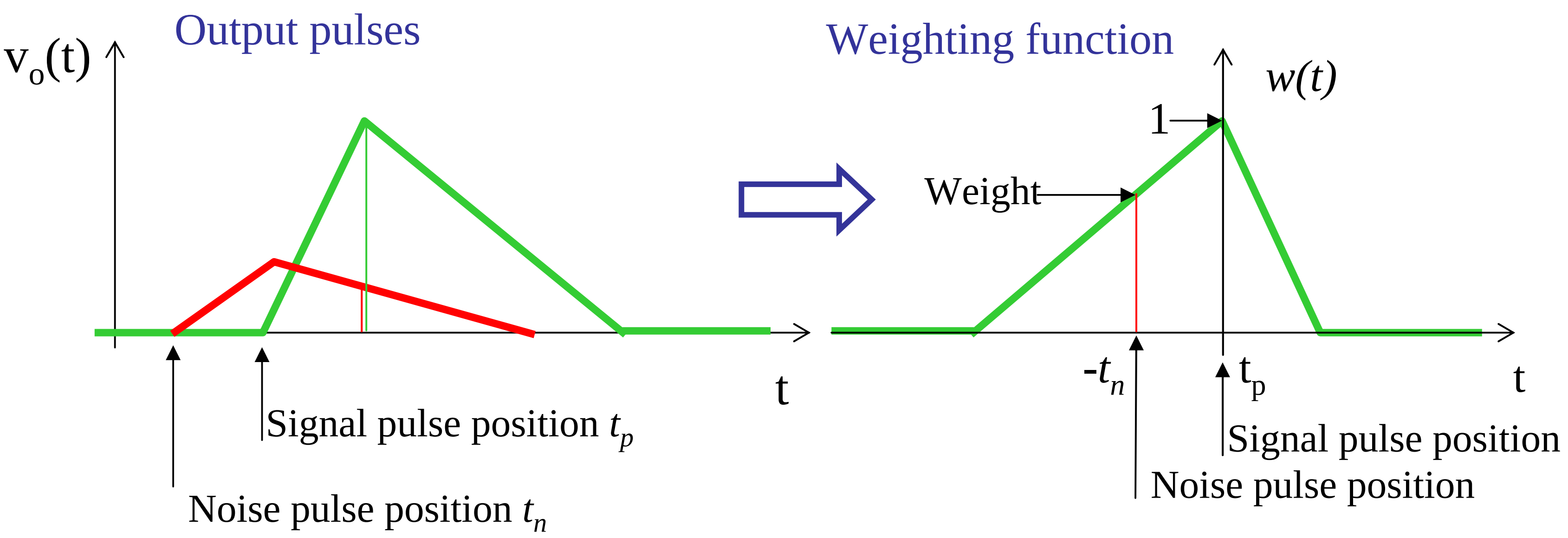




Very rough approximation

The weighting function

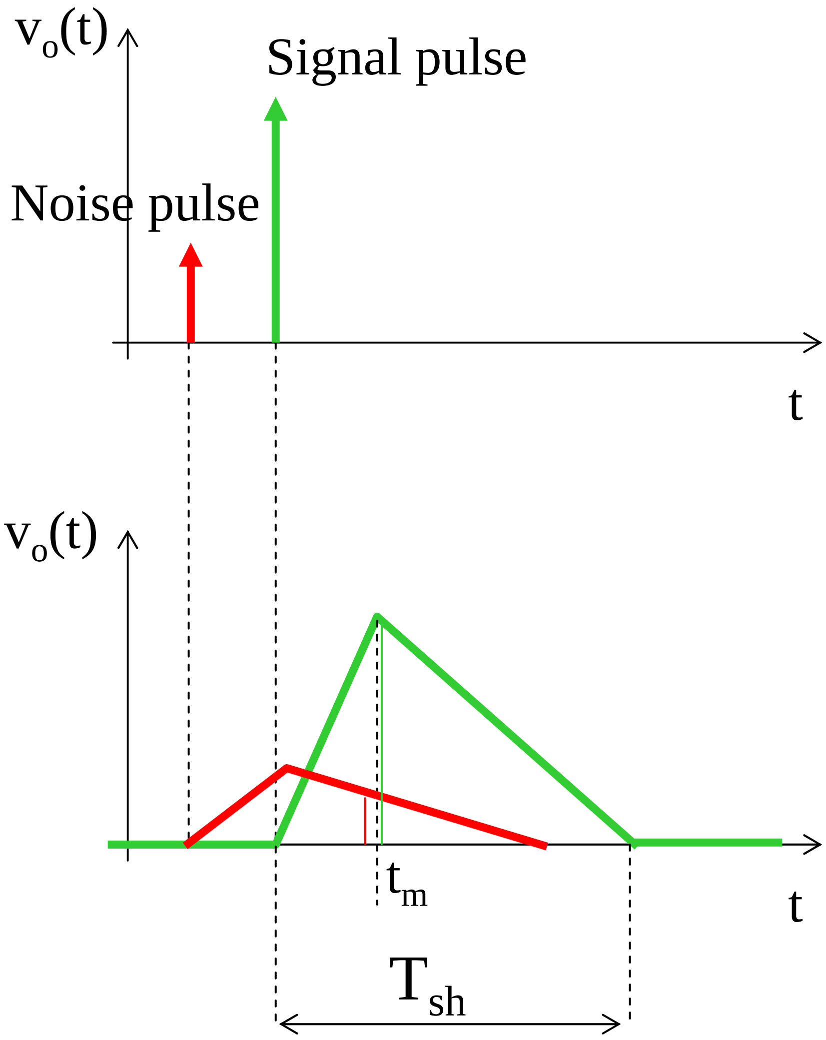
The concept of the ‘weighting function’



Current signal pulses
and current noise pulses
produces at the output
waveforms of the same shape.

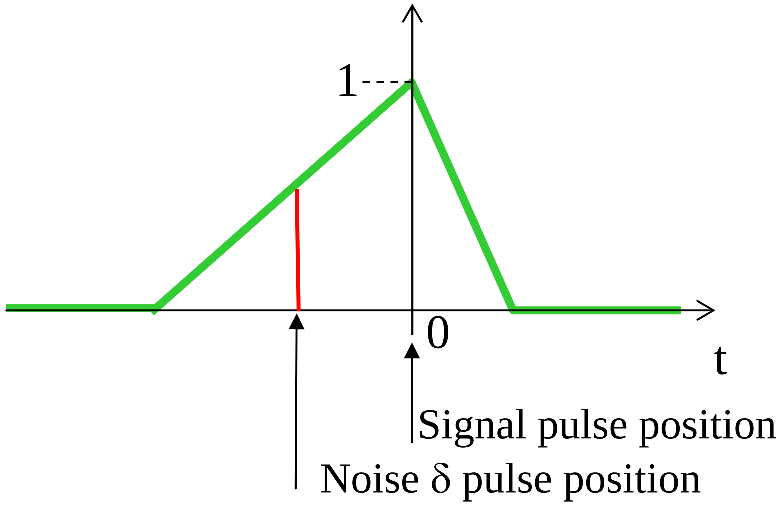
The ‘weighting function’ *w(t)*,
which is the ‘time-reversal’ of the output waveform,
gives the weight with which an input noise pulse
contributes to the peak amplitude of the output signal,
as a function of its displacement from the input signal pulse.

Noise effects – δ pulses

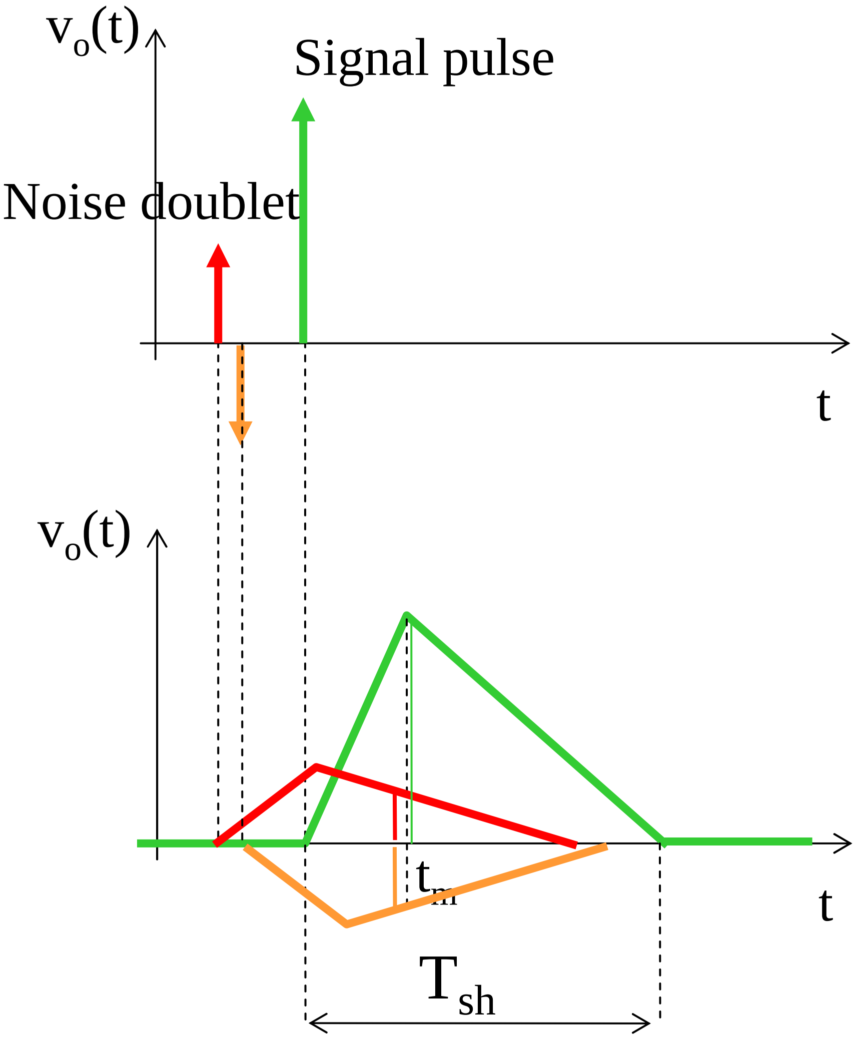


The measurement of the peak amplitude of the output signal pulse is affected by errors due to the superposition with the random output noise pulses.

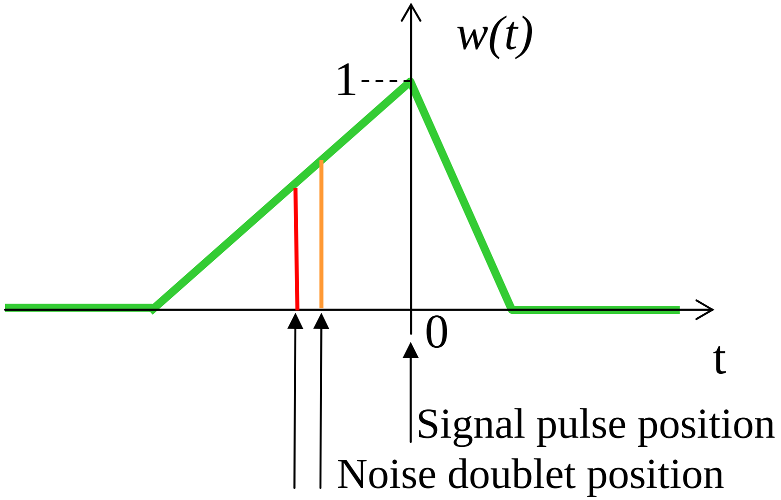
The contribution of a given noise pulse is determined by the value of the weighting function evaluated at the time $-t_n$ corresponding to the time occurrence of the noise pulse with respect to the signal pulse.



Noise effects – doublets of δ pulses



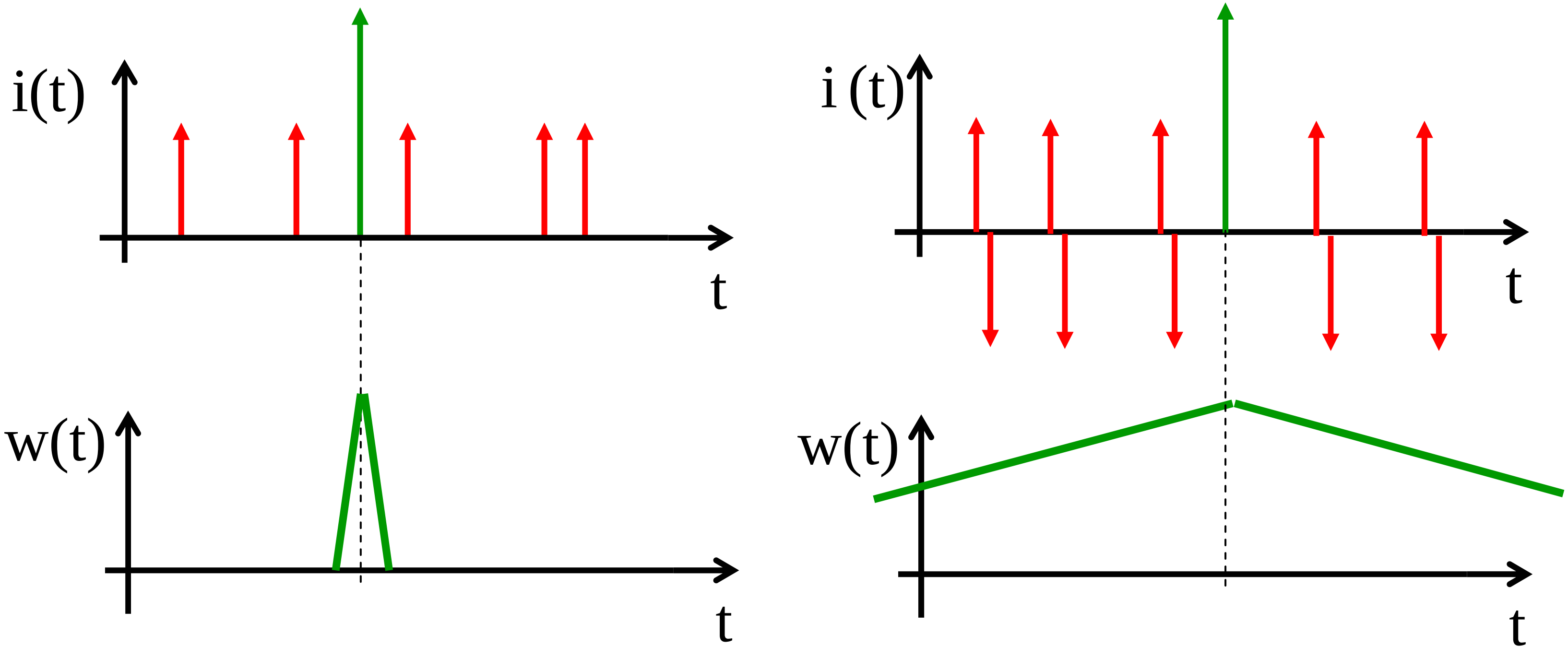
The contribution to the peak amplitude
of the output signal
of the two δ of the doublet
is slightly different
because of the slightly different value
of the weighting function
at the respective position of the two δ.



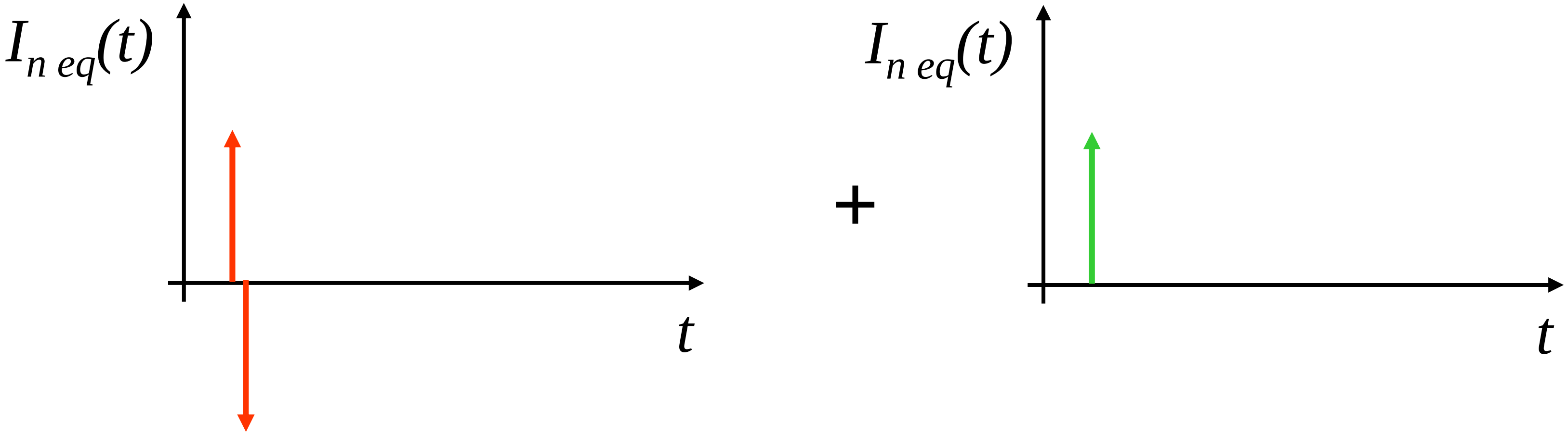
How to choose the shaping time
White noise - Intuitive considerations

a) Parallel noise (δ current pulses):
the weighting function should be as short as possible in order to collect contributions from the lowest possible number of δ pulses.

b) Series noise (doublets of δ pulses):
the weighting function should be as long as possible (tails with small slope) in order to weight as much as possible equally the two δ pulses of the doublets.



How choose the shaping time
1/f noise - Intuitive considerations



The δ-doublet would require
long shaping time

The δ would require
short shaping time

The opposite requirements makes
the 1/f contribution
independent of the shaping time